Problem Set 4 - LV 141.246 QISS - 30.4.2012

1. Universal Two Qubit Gate

$$\sqrt{\text{SWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Show that the gate $\sqrt{\text{SWAP}}$ is a universal two-qubit gate by building a CNOT out of one or several $\sqrt{\text{SWAP}}$ and single qubit operations.

The SWAP gate is not a universal. Can you find an argument.

2. Two Qubit Interactions

Consider the following two qubit interaction Hamiltonians and study unitary evolution. For how long do you have to apply these interactions in order to create a universal gate.

(a) Ising interaction

$$H_{zz} = -j_{zz}\sigma_{z,1}\sigma_{z,2}$$

(b) Heisenberg interaction

$$H_{jj} = -j_{jj}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2} + \sigma_{z,1}\sigma_{z,2})$$

(c) XY interaction

$$H_{xy} = -j_{xy}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2})$$

3. Grover Algorithm

Implement the Grover algorithm in MATLAB on a three qubit register. Implement your oracle such that 5 (binary 101) is the sought-after entry.

$$f(101) = 1 \qquad f(x) = 0 \text{ for } x \neq 101$$

For an explanation of the Grover algorithm, please refer to Nielsen Chuang, *Quantum Computation and Quantum Information*,pp 248ff.

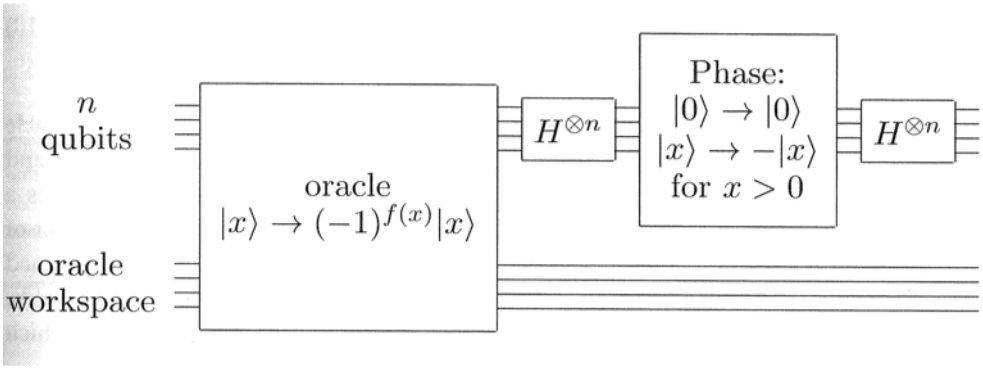

Abbildung 1: Grover Gate

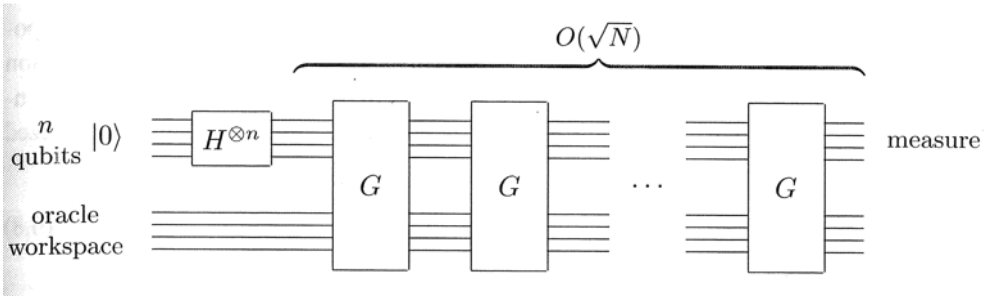

Abbildung 2: Grover Algorithm